Fund Management assignment - Investment strategy for Polish Market

SID: 829283

1. Introduction

Modern Portfolio Theory is a fundamental mathematical framework in fund management. Based on assumption, that assets are characterised by expected return and volatility, it is a method of finding optimal allocation (weights) of capital into those assets. Rational investor is interested in maximizing profits and minimizing risk. In a pool of financial instruments, which differ in terms of risk, volatility and correlation between each other, there exists only one set of weights, which provide desired, by specific investor, risk to reward ratio. This is named Optimal Risky Portfolio lying on opportunity set curve, which is nothing else than pareto-optimal set of solution for multi objective optimization problem (Figure 1).

The task for this assignment was to advice an investor with risk aversion equal to 3, how to allocate his funds, so he can expect best return on investment, with minimum risk possible – maximum he is allowed to undertake. This set will be called – Optimal Complete Portfolio, as presented on Figure 1. Whole process will be described as follows, starting with initial characteristics of data.

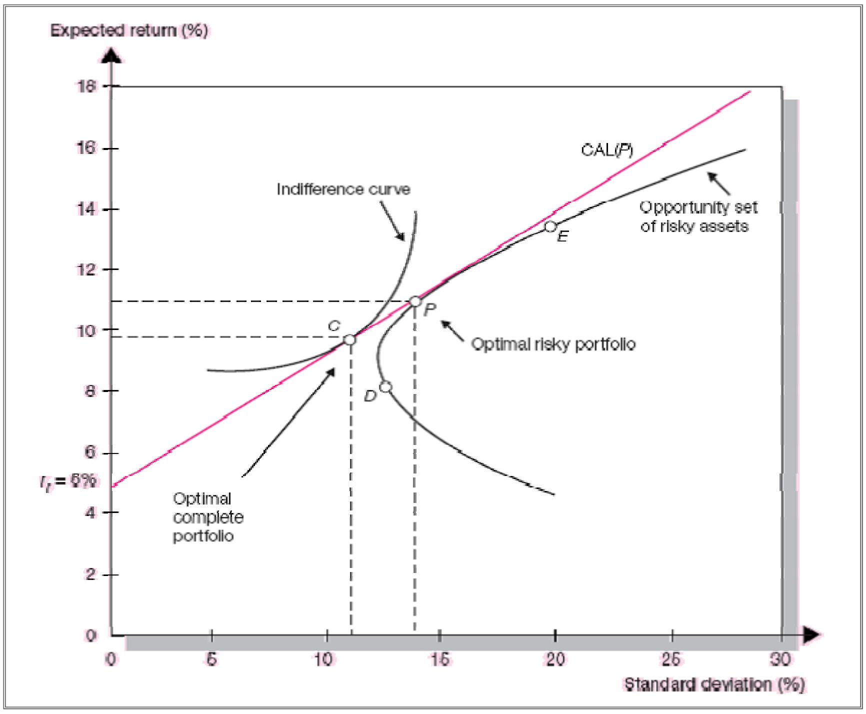


Figure 1. Overall portfolio (Bodie-Kane-Marcus, 2005)

2. Data description

In order to generate efficient frontier, input data were prepared performing following steps:

- Decision about the Market to analyze.
  Constituents of 3 main Polish Stock Market indices have been initially chosen
  - WIG20 – 20 large Cap shares
  - MWig40 – 40 Medium Cap shares
  - SWig80 – 80 Small Cap Shares
- Time frame
  Series of 2500 daily prices for each share have been prepared using (Datastream2008).
- Illiquid shares.
  Not every asset in the created set was liquid enough. Secondly, not all of the shares had their IPO's five years ago. Those, which did not meet above criteria have been excluded. This has been done by analyzing percentage of days traded out of all of available data. The limit was set at 90% of days traded over past 250 days. As a result, 37 shares have been kept for further analysis.
- Stationarity.
  As required by the theory, data should be stationary, what is not the case for prices. Therefore closing prices were converted into logarithm returns (expressed in percentage).
- Normality
  Returns have been tested for normality using Jarque-Bera test. Table 3 in Appendix contains results for normality test for each of the asset. Unfortunately, Jarque-Berra test shows that none of considered returns are normally distributed. One reason for this is the fact, that estimation period might be not long enough and it was possible that included mostly returns of bullish market. Jarque-Berra test is asymptothic, therefore additionally, Lilien Test (Conover, 1980) has been performed, as it is designed for small samples. The results confirmed J-B test, that data are not normally distributed.
- Expected returns, volatility and correlation estimates
  For the purpose of this assignment, simple equally weighted methods have been employed.
  - Mean (Average) value of past returns – Expected Returns
  - Root square of Variance for the period - Volatility

  Obtained values have been used as future expectations, but have been further annualized using following properties:
    - $a_{annualized} = 250_{trading\ days} \cdot \mu_{daily}$
    - $\sigma_{annualized} = \sqrt{250_{trading\ days}} \cdot \sigma_{daily}$

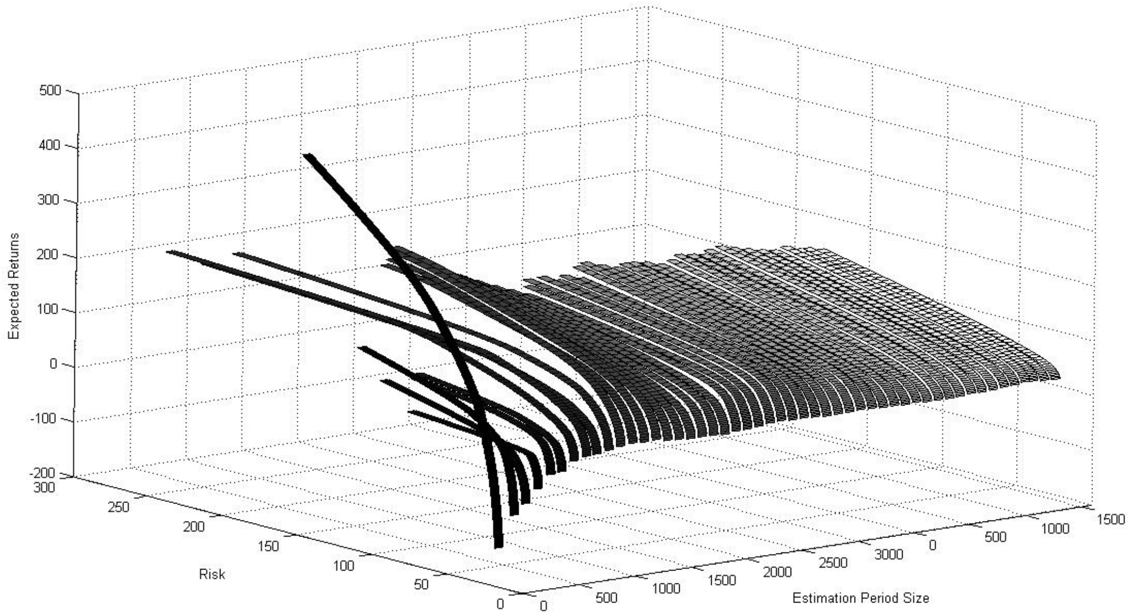
  Further in this report, analysis will be performed showing how estimation period length for volatility introduced variability into results.

3. Experiments

After successful data preparation analysis has been performed. It's goal was to find the best set of parameters which will result in best portfolio possible portfolio.

Varying time frame for variance estimation

In this part, different time frames for variance estimation have been chosen. This was to illustrate how, choosing inadequate period may result in varying portfolio performance.



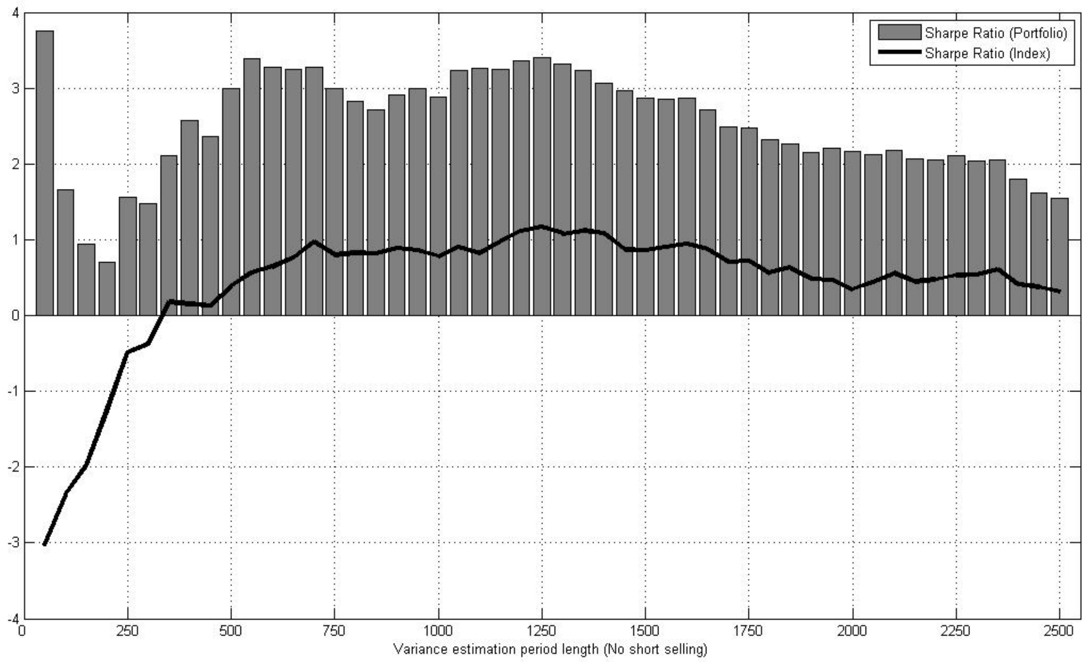


Figure 2. Sharpe Ratio as a function of estimation period length (No short selling)

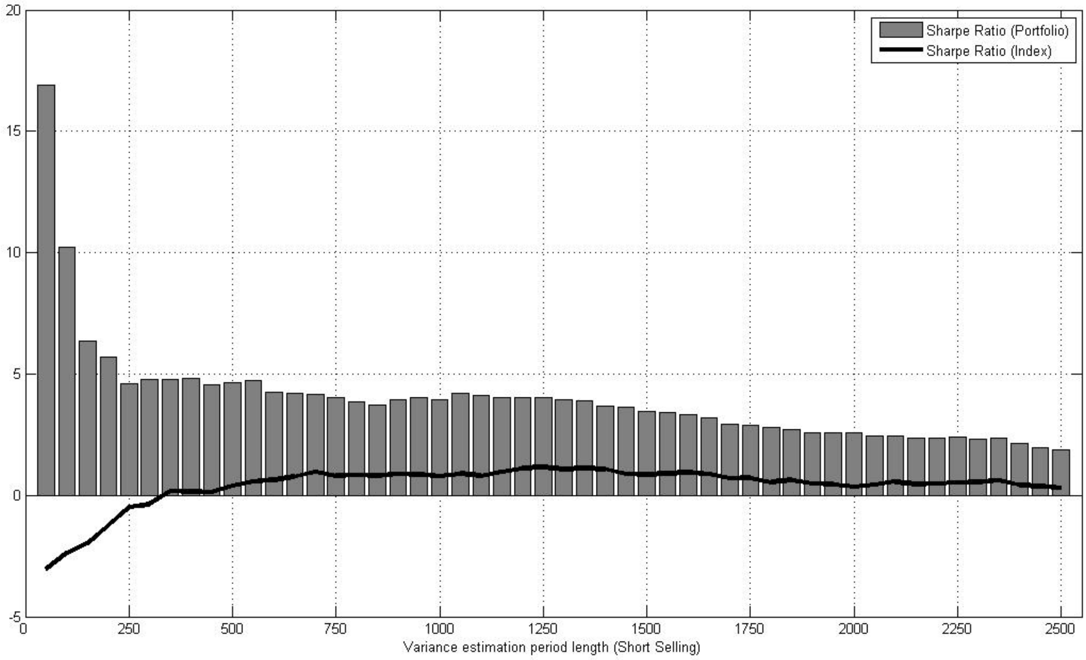


Figure 3.Sharpe Ratio as a function of estimation period length (With short selling)

High return at short estimation period is a result of poor estimation of Mean and Variance values.

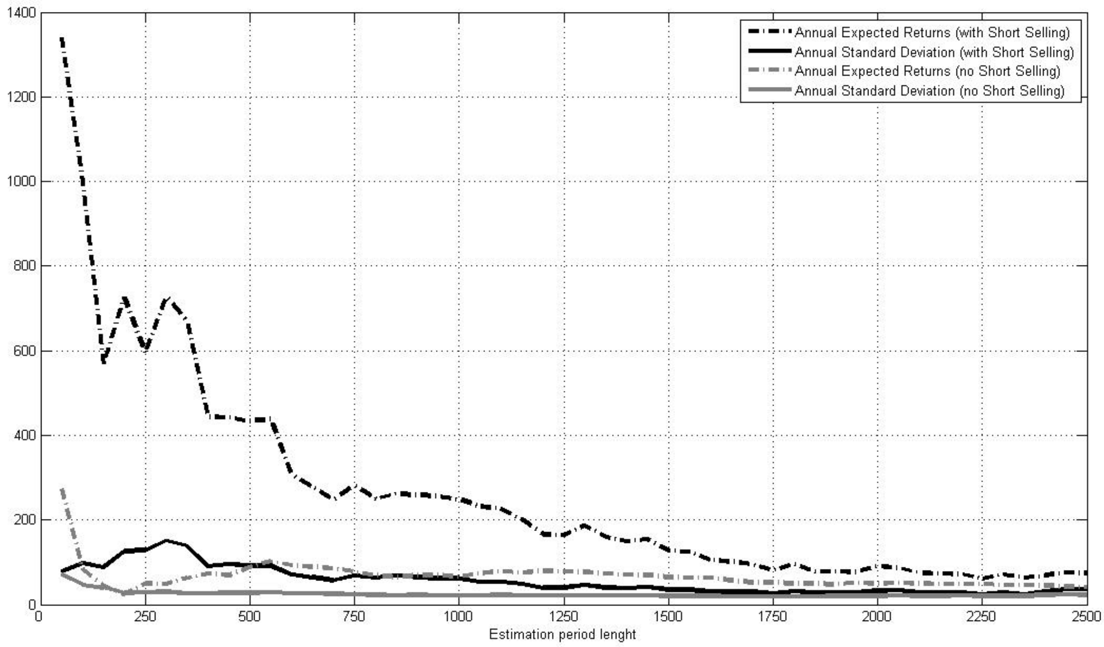


Figure 4. Expected Returns and Volatility of Optimal Risky Portfolio with respect to estimation length

Figure 4 presents relationship of Expected Returns and Standard Deviation for Optimal Risky Portfolio with respect to estimation period size. All of the values measured converge while estimation period size grows. This is an example showing how easy it is, by using inappropriate estimation period length, to under or over estimate those parameters. Analyzing Figure 4 one can notice that when estimating portfolio using short estimation period, expected return is higher than the true value. By using longer estimation period, it is possible to achieve better estimations. There is always a trade-off between choosing too long and too short estimation period length. On one side, too long estimation period will not react quickly enough for unexpected conditions. On the others side, too short estimation period is likely to cause high fluctuations in efficient frontier, which will cause higher number of transactions – therefore higher cost. It is important to notice that this model does not include cost of purchasing stocks, which is significant factor in fund management. Based on Figure 4, Figure 3 and Figure 2 – two different estimation periods have been chosen for creating optimal risky portfolio: 500 days and 1250 days. Those values seem to be a good choice from a Sharpe Ratio point of view as well as their mean variance.

Investment strategy preparation and justification

Based on analysis in previous section, two estimation periods length have been chosen for further analysis: 500 days and 1250 days. Efficient frontier and weights allocation were obtained using, written for the purpose of this assignment, Matlab script (Appendix 1).

Risky Assets only

Table 1. Optimal Risky Portfolio presents results obtained for optimal risky portfolio (assuming $R_f = 5\%$). As clearly presented, short selling strategy is able to generate better returns than long only. In both cases, index is outperformed (comparing to Index's Sharpe Raio). Assets allocation for risky only portfolio has been presented graphically in Figure 6 and Figure 8.

Table 1. Optimal Risky Portfolio

	Estimation period length	Expected Return	Risk	Sharpe Ratio	Index Sharpe Ratio
Short selling Allowed	500	433.6	91.9	4.67	0.38
	1250	164.3	39.5	4.03	1.16
No Short Selling	500	91.9	29.0	2.99	0.39
	1250	80.1	22.3	3.39	1.16

Risky and Risk free asset

In general, when designing strategy for investment with risk free assets, risk aversion of investor has to be known. This kind of strategy limits to a decision about how much to invest into risk free assets and how much into optimal risky portfolio, treating them as a portfolio of two assets. It is crucial to maximize investors utility with respect to her risk aversion. This is done by calculating y* - fraction of Risky Assets in portfolio. In these cases, this value is larger than 1, what suggests to borrow at risk free rate and invest borrowed money. Here, it was assumed that borrowing rate is equal to risk free rate, what is not the case in real world. This can be easily corrected, and will only result in lower complete portfolio return. It is important to notice, that expected returns are very high. This is not very unusual, because suggested strategy makes use of leverage (borrowing to invest).

Table 2. Optimal Risky Portfolio with Risk-free Asset

	Estimation period length	Complete Exp Return	Complete Risk	Sharpe Ratio	Index Sharpe Ratio	Fraction of Risky Assets
Short selling Allowed	500	740.4	157.6	4.67	0.38	1.69
	1250	546.6	134.3	4.03	1.16	3.40
No Short Selling	500	299.5	98.3	2.99	0.39	3.44
	1250	392.9	114.1	3.39	1.16	5.08

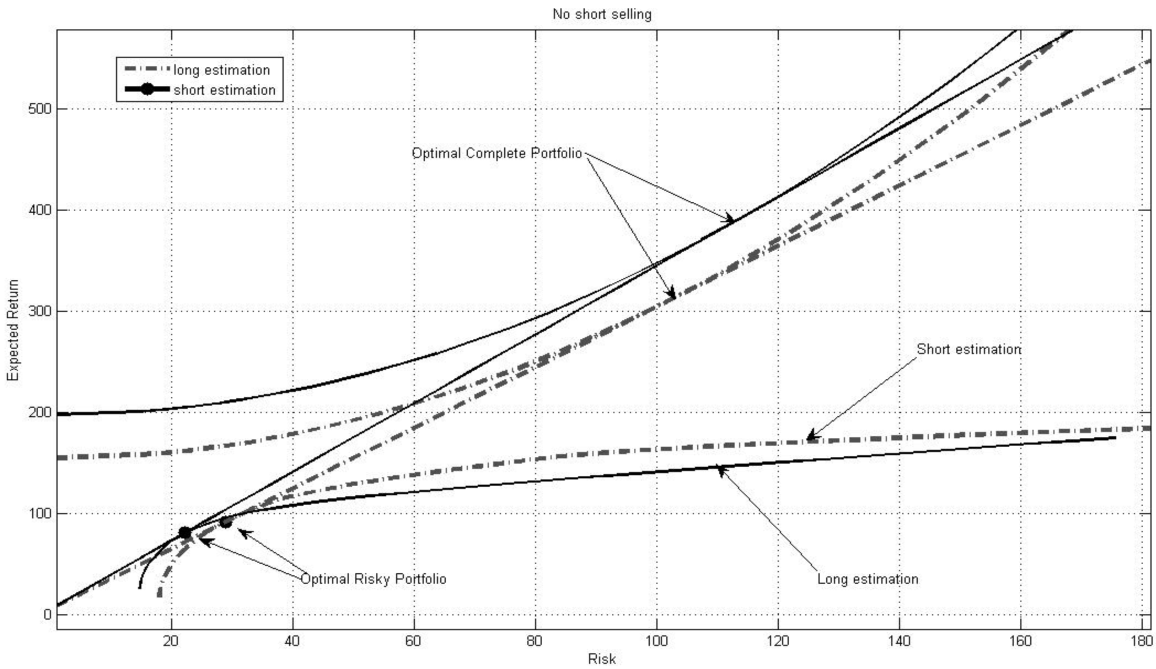


Figure 5. Efficient frontier (no short selling)

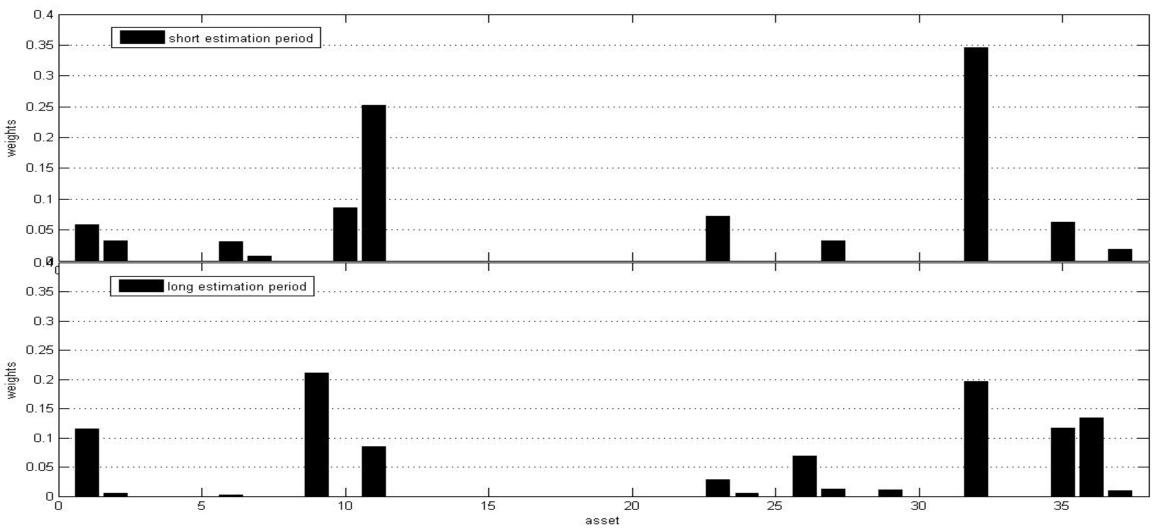


Figure 6. Optimal Risky Portfolio assets allocation (no short selling)

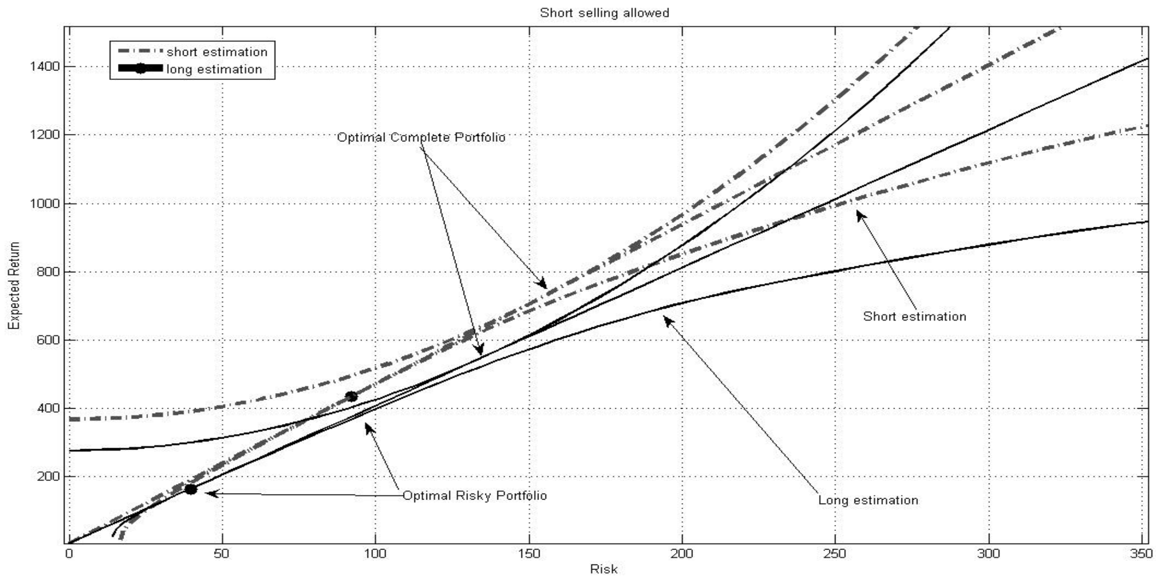


Figure 7. Efficient frontier (short selling allowed)

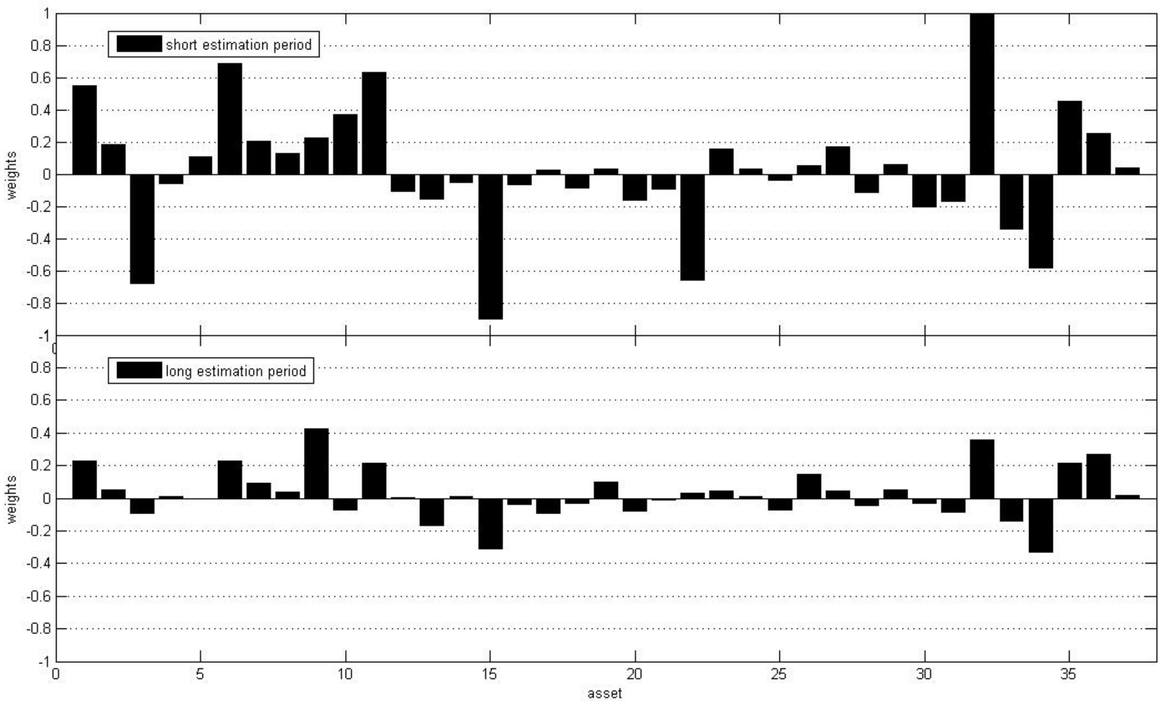


Figure 8. Optimal Risky Portfolio assets allocation (short selling allowed)

Conclusion and discussion

MPT is a formal strategy for obtaining best assets allocation, theoretically sound, but has amazing number of assumptions and therefore adequacy to the real world is questioned. First of all, method is very sensible for estimates of variance, covariance and mean value for each of the stock. Although optimization method can always produce solution, it is the quality of those initial estimates that drive overall results. One has to keep in mind, that method is indeed a representation ideal not a real world conditions – exclude transaction costs, divisibility of stocks (i.e. one can but 1 or 2 stocks instead o 1.3). Thus many obstacles make this method not very popular throughout fund managers (as cited during lecture about MPT) and they prefer to use momentum strategies.

Nevertheless, after experimenting with this method, I can conclude that it is very promising. Its formal soundness is visible at each step of calculations. It provides answer for so many questions, i.e. how to diversify portfolio, how to achieve best return for minimum risk, given investor's risk aversion, how to make use of risk free asset, etc.

One only has to be aware of mentioned limitations and an effort to use best available solution, minimizing impact of them. This could be, i.e. using other algorithm for estimating mean and variance (EWMA, ARCH, GARCH, OCLH). Additionally, instead of using pure past data, one could focus on using forecasts as an input for the model.

In general, Markovitz Portfolio Optimization, could be implemented as constantly running algorithm, which could produce best advice at any given point in time. Unfortunately, from my preliminary experiments, one problem regarding this idea is that often for small changes in estimates, large changes in weights values occur, suggesting that holding of one stock should be almost fully replaced with other stock. This as a result provide large transaction costs and need to be improved. Ideally, method should take into account mentioned transaction costs, especially on the level of optimization procedure.

Finally, I have observed that Polish market, ignoring mentioned errors of the method, provides great opportunities for investment and if the results are true indeed, it is very tempting to build a real portfolio based on obtained results. However, better understanding of the method and incorporation of error minimization strategies is highly recommended before commencing this investment. In this report, high returns were obtained deliberately – by choosing estimation period which produce highest Sharpe's ratio. This might not be a good idea in real world. It would be better to estimate weights for a period which produces lowest Sharpe's ratio. This is because in investment world, it is better to assume the worst that may happen, rather than being overoptimistic.

Bibliography

Bodie-Kane-Marcus. (2005). *Investments, 5th edition.* McGraw Hill.

Conover, W. J. (1980). *Practical Nonparametric Statistics.* Wiley.

MathWorks Inc. (2007). Matlab Help File.

Appendix

Table 3. Test for Normality results

Hypothesis	Probability	Test Statistics	Kurtosis	Skewness
1	0	1016	6.0	0.4
1	0	90416	31.8	2.8
1	0	15153	15.0	-0.5
1	0	10545	12.9	0.9
1	0	48864	24.6	0.1
1	0	889	5.9	0.1
1	0	3178	8.4	0.6
1	0	117858	36.1	2.5
1	0	6369	10.8	-0.1
1	0	2377	7.7	0.2
1	0	2440	7.7	0.5
1	0	80825	30.4	2.3
1	0	6326	10.8	0.0
1	0	16963	15.5	1.2
1	0	3913	9.1	0.3
1	0	63652	27.6	0.4
1	0	32986	20.7	-0.3
1	0	4594	9.6	0.1
1	0	1060	6.1	0.3
1	0	2722	8.1	-0.1
1	0	323928	58.5	1.3
1	0	3877	8.9	0.8
1	0	124033	36.6	3.7
1	0	4839998	217.3	7.5
1	0	9473	12.2	1.1
1	0	10382	12.8	0.8
1	0	298422	56.0	3.1
1	0	11562	13.2	1.3
1	0	712994	85.4	1.2
1	0	10379903	316.9	-10.3
1	0	9043	12.0	1.2
1	0	5230	9.8	1.0
1	0	28781	19.5	-0.7
1	0	3915	9.0	0.6
1	0	7929859	277.8	9.4
1	0	21750	17.1	1.4
1	0	235467517	1499.6	33.6

Listing 1. Matlab script for Portfolio Optimization

```
clear all;
 load wholeData.mat;
 dates = x2mdate( dates );
 dataRet = tick2ret( data, [], 'Continuous' );
 indDataRet = tick2ret( dataInd );
 dates = dates( 2 : end, 1);
%%
numEffObs = 500;
[ExpReturn, ExpCovariance, NumEffObs] = ewstats( dataRet( end - numEffObs + 1 : end, : ));
anNum = find( isfinite( ExpReturn ) );
dataRet = dataRet( : , anNum );
numOfAssets = size( anNum, 2 );
colheaders = colheaders( :, anNum );
%%
portfER = [];
portfRI = [];

hold on
shrpRat = [];
indShrpRat = [];
portRetGl = [];
portRiGl = [];
% figure; hold on;
k=1;
final = {};
% for i = [ 500 1250]
    i=500
    figure
%% start analysis from here

obsPer = i;

expRet = mean( dataRet( end - obsPer + 1 : end, : ) ) * 250*100;
stdDev = std( dataRet( end - obsPer + 1 : end, : ) ) * sqrt(250)*100;
indEr = mean( indDataRet( end - obsPer + 1 : end, : ) ) *250*100;
indStd = std( indDataRet( end - obsPer + 1 : end, : ) ) * sqrt(250)*100;

ExpCovariance = corr2cov( stdDev, corr( dataRet( end - obsPer + 1 : end, : ) ) );
PortWts = ones( 1, numOfAssets ) / numOfAssets;

[PortRisk, PortReturn] = portstats(expRet, ExpCovariance, PortWts);

%%
%fminunc
%multiobjective optimization
NumPorts = 50;
```

```
[PortRiskO, PortReturnO, PortWts] = frontcon(expRet, ExpCovariance, NumPorts, [], [-
1.3*ones(1,numOfAssets);1.3*ones(1,numOfAssets)]);
%%
%hold on
portfER = [ portfER PortReturnO];
portfRI = [ portfRI PortRiskO];
RisklessRate = 5;
%%
sr = 1;
se = 1;
[slope,I]=max((PortReturnO-RisklessRate)./(PortRiskO));
RiskyRisk=PortRiskO(I);
RiskyReturn = PortReturnO(I);
RiskyWts=PortWts(I,:);
%%
i
shrpRat = [ shrpRat (PortReturnO( I ) - RisklessRate) / PortRiskO( I ) ];
indShrpRat = [indShrpRat ( indEr( 1 ) - RisklessRate) / indStd( 1 )];
portRetGl = [ portRetGl RiskyReturn ];
portRiGl = [portRiGl RiskyRisk ];



 A= 3;
 plot (PortRiskO*sr, PortReturnO*se); hold on
 plot(RiskyRisk*sr, RiskyReturn*se, '*r')
 plot([0; PortRiskO*sr], [RisklessRate; slope*PortRiskO*se+RisklessRate])
 U = RiskyReturn - 0.005*A*RiskyRisk;
 y= (RiskyReturn-RisklessRate)/(0.01*A*RiskyRisk^2);
 ymax = y;
 U= RisklessRate + y * (RiskyReturn - RisklessRate) - 0.005*A*(y^2)*(RiskyRisk^2);
 Uer = U + 0.005 * A * PortRiskO.^2;
 plot([0; PortRiskO], [ U; Uer ]);
%%
[Umax, Imax] = min(Uer./PortRiskO);
CplRisk = PortRiskO(Imax);
CplReturn = Uer(Imax);

plot(CplRisk, CplReturn, '*r')

%%
Uw = [];
for y = 0.05:0.05:5
   Uw = [Uw RisklessRate + y * (RiskyReturn - RisklessRate) - 0.005*A*(y^2)*(RiskyRisk^2)];
end

cutoff = 0.01;

final(k,:)=colheaders;
k=k+1;
```

```
final(k,:) = num2cell(RiskyWts)
k=k+1;
RiskyRisk
RiskyReturn
CplRisk
CplReturn
ymax
shrpRatio = (RiskyReturn-RisklessRate)/RiskyRisk
shrpRatioCpl = (CplReturn-RisklessRate)/CplRisk
indShrpRatio = ( indEr( 1 ) - RisklessRate) / indStd( 1 )
% end

%%
jbres = [];
lilres = [];
lilrt = [0 0];
for i = 1:37
   [jbhy, jbp ]  = jbtest(dataRet(:,i));
   jbres = [jbres; [jbhy jbp]];
   lilrt(:) = lillietest(dataRet(:,i));
   lilres = [ lilres; lilrt];
end
```