Risk Management –VaR estimation

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1. Introduction

Value at risk (VaR) is a general concept that has broad applications to measure the market risk of portfolios of assets. It is applied in finance to manage many types of risk. VaR is the worst expected loss which will not be exceeded with a given confidence level over a given time period. There are three commonly used methods for evaluating VaR: historical method, variance-covariance method and Monte Carlo simulation.

This report presents estimations of VaR using all of those methods for a portfolio of 3 stocks: British Petroleum plc (200 shares), British Airways (100 shares) and Barclays plc (150 shares). Additionally, simple sensitivity analysis has been performed at different confidence levels (95% and 99%) and estimation period length: last 7 years, the most recent 1000 prices, and the most recent 250 prices. This portfolio, at 5th of March 2008 was valued at £2030.25 (203025 pence).

Finally, a discussion will be performed about the procedures of each method, the assumption behind each analysis, obtained results, differences we found between methods and sensitivity of VaR to the various samples sizes.

2. Historical Simulation

Assumptions

In this method we assume that changes in the market from day to day are the same as the changes that took place in the past. In other words, history (in the scene of changes of the market or a stock) repeats itself. Another assumption is that there is no limit to the type of a distribution as it is non parametric method.

Procedure

The objective of the historical simulation method is to find a large number of possible outcomes for future assets' prices. Using today's prices, we can construct the distribution, that will help us estimate any risk measure i.e. VaR

To do so, first we calculated daily returns of the three stocks for 7years, 1000 days, and 250 days. Second we calculated the change of portfolio value for each day of the 7 years, 1000 days, and 250 days; assuming the individual portfolio value remains intact. Third we arranged these values in descending order (from highest to lowest). Finally the value of the portfolio that has the 1% (5%) of the lower returns in the tail is taken to be the estimated Var for the next day at 99% (95%) confidence interval.

Note: in our results we could not read off the exact value of the portfolio that had the 1% (5%) of the lower returns to estimate the Var, therefore we used ***linear interpolation*** to find these exact values; by using the closest value above, and closest value below 1%(5%), and substituting them into the equation:

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{x_2 - x_1}$$

Where:

y= portfolio value at 5th percentile

Results

1. Using last 7 years of daily prices we can estimate with 99% confidence that in a day we can lose no more than £***131.18*** pence and with 95% confidence we can lose no more than £***52.87*** *pence* in a day.

2. Using the most recent 1000 prices we are 99% confidence that in a day we can lose no more than £***64.06*** and are 95% confidence that in a day we can lose no more than £***35.69***.

3. Using the last 250 prices we can say with 99% confidence that in a day we can lose no more than £***89.53*** and with 95% confidence that in a day we can lose no more than £***56.66***.

Table 1. Historical Method - results

	95%	99%
7yrs	52.87	131.18
1000 days	35.69	64.06
250 days	89.53	56.66

Note: looking at the results of the 99% confidence and the 95% confidence we can see that the value at risk at the 99% is always greater than those at the 95% confidence, this makes sense because to increase your confidence level to 99% and therefore reduce your % error to 1%, the value at risk should be high enough to account for any possibility of loss.

Analysis of results

With the Historical simulation accurate percentile estimates of the VaR (e.g. 5th percentile) require data to be accurate. Traditionally, the longer the data series used, the more accurate the results for VaR, but to long estimation period – less recent the VaR is.

However, as can be seen from our results, VaR estimations for the most recent 250 prices are greater than that of the most recent 1000 prices, which are less than those of the last 7yrs. As volatility in stock prices caused by sporadic events have an effect on estimated VaR, these typical results could be as a result of such.

The 250 price returns are most likely more affected by the recent credit crunch crisis than the 1000 price returns, thus it is most likely biased, and hence its results. Similarly, the 7year data is more likely to be exposed to more “price affecting” events, such as unusual events like the September 11 terrorist attacks in 2001 which especially affected airline prices as depicted by the huge volatility in British Airways returns during that period. This gives room to question the accuracy of the data in depicting reoccurring history, as both instances can lead to the historic simulation overestimating the exposed risk.

3. The Variance-Covariance method

The Variance-Covariance method assumes that returns on stock are normally distributed. This has been tested using Jarque-Berra test for normality. Unfortunatelly, none of returns are normally distributed, because with probability lower than 0.05 null hypothesis of J-B test can be rejected. Although, we assumed normality of returns as it is major assumption. V-C method requires only two factors which are an average return μ (or expected) and a standard deviation (σ) to be estimated. Firstly, estimate an average return and a standard deviation σ.

$$\omega_i = V_i / V_p$$

$$\mu_p = \sum_{i=1}^{N} \omega_i \mu_i,$$

$$\sigma_p = \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}}$$

Where: V - beginning value (in currency units)

i - refers to "of asset I"

p - refers to "of the portfolio"

N - Number of asset

$\boldsymbol{\omega}$ - refers to a vector of all ω_i

$\boldsymbol{\Sigma}$ = covariance matrix

Then, a normal distribution curve can be plotted and the worst 5% and 1% would automatically lie on the curve. After that, desired level of confidence is chosen and VaR calculated by multiplying the standard deviation obtained with a value from the selected confidence level.

Confident level	VaR
95%	-1.65 x σ
99%	-2.33 x σ

Assumption

It is assumed that all market factors follow a multivariate normal distribution. Additionally, calculated mean, variance and correlation are unbiased estimates.

Analysis of result

For the last 7 years, with 95% and 99% confidence, the maximum daily loss will not exceed £65.75 and £92.84 respectively.

For the most recent 1000 prices, with 95% and 99% confidence, the worst daily loss will not exceed £48.42 and £68.37 respectively.

For the most recent 250 prices, with 95% and 99% confidence, the maximum daily loss will not exceed £71.50 and £100.97 respectively.

Table 2. Variance Covariance - results

	95%	99%
7yrs	65.75	92.84
1000 days	48.42	68.37
250 days	71.50	100.97

In accordance with the VaR's estimated, it is obvious that the sensitivity is not affected by sample sizes. The worst expected loss for the most recent 250 prices is the largest while the most recent 1000 price shows the smallest maximum expected loss. Generally, because of the credit crunch and turmoil in the financial markets, most recent 250 prices should have the expected loss more than 1000 prices and 7 years price and this is the case indeed.

It is important to take into account when the variance-covariance method is used particularly the fact that some market price movements might not be normally distributed. They could not have normal distribution but they may tend to have a somewhat more frequent circumstance of extreme observations, called heavy tails. Moreover, as a result of some unexpected situations, this method could sometimes not represent market risk properly. In addition, it could be the case that the precedent does not always give a good direct for the future.

4. Monte Carlo Simulation

Monte Carlo simulation is a method, which is based on "artificial" increase of available sample size. This is not strictly a method for VaR calculation but is a framework which is based on utilization of random number generator in order to simulate possible outcomes. At first, variance, mean and correlation is estimated based on historical data. Based on those estimates, random, normally distributed (with estimated mean and variance) sample is generated. Finally, random series is transformed using Cholesky decomposition, to produce random correlated sample. This is further processed in the same way as in historical method.

Assumptions

Main assumption is the fact that data follow some parametric distribution and that those parameters can be estimated.

Analysis of Results

Table 3. VaR - Monte Carlo Simulation - results

	95%	99%
7yrs	133.94	145.67
1000 days	97.14	105.75
250 days	134.19	145.34

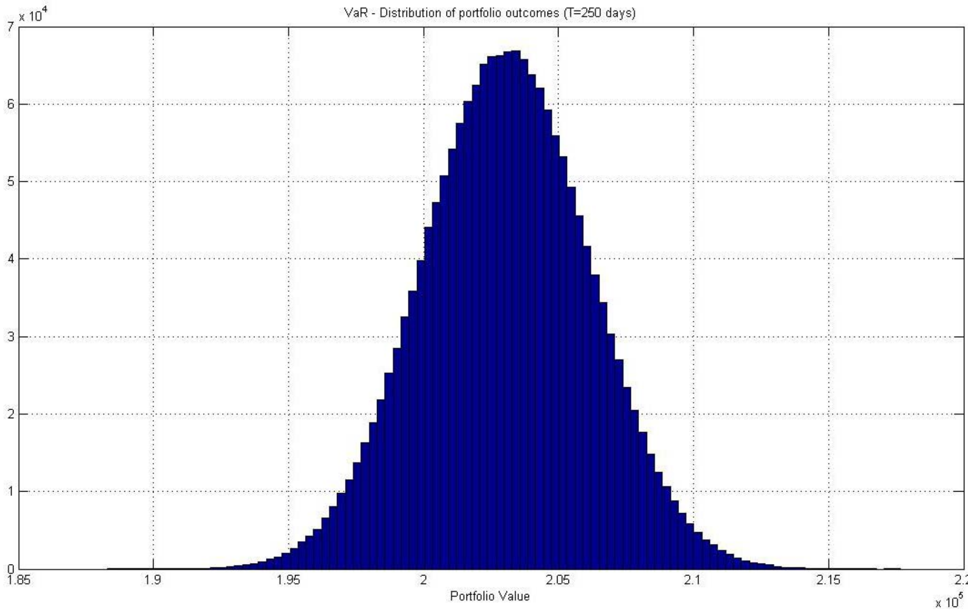

Figure 1. Sample outcome of simulation (T=250 days)

Monte Carlo simulation results are larger than provided by any other method. There are some issues with this method and those could have influenced these results. One is undoubtedly estimation of parameters of Normal distribution (assuming that returns are normally distributed). These estimates may be far from real current value of volatility or returns. Going further, generation of random simulated correlated returns is also biased, especially if sample is too small. But on the other side, MC results might be closer to real VaR. It is based on large sample thus have higher statistical significance than both remaining methods.

5. Differences/Comparison between methods

Both methods produced roughly similar results. It is documented[1] that parametric VaR estimation approach tends to underestimate actual losses for cut-off points below 2% and over estimate risk at the 95% confidence level as compared to the historic simulation method. This is true given our results for the 95% confidence level, but only the most recent 1000 prices has the V-C VaR at 99% confidence lower than the HS VaR. It can also be seen that the VaR for both methods are closest for the most recent 1000 prices data at both confidence levels.

Differences continued:

- Historical analysis is non-parametrical method, thus it is not required to know distribution of data to perform this analysis, opposite to V-C method, which is based on estimates of distribution parameters.
- Disadvantage of historical method is a lack of data, on which the results are based, and this can be overcome by applying Monte Carlo method where large sample of data can be generated using stochastic random number generator,
- The methods differ in their ability to capture the risks of options and option-like instruments. While the historic simulation can work well, the variance-covariance method is less able to capture the risks of options and option-like instruments as it may not adequately capture how the option changes with changes in the underlying prices.

- The historic simulation method is computationally easy to implement, although cumbersome with a huge amount of data, as long as past values of the data are available. The variance-covariance method is also easy to implement, but becomes moderately difficult depending on the complexity of the instruments in the portfolio, and availability of data.
- The historic simulation is easier to explain to senior management or anyone with limited knowledge of the subject area than the variance-covariance method.
- There is no flexibility in analyzing the effects of changes in the assumptions of historic data as it is directly tied to historical changes in the basic market function. However, the variance-covariance method is more flexible in incorporating alternative assumptions about correlation or standard deviation, but not in the statistical distribution of market data.
- Both methods don't give reliable estimates of VaR when historic data is atypical. However, this is more so for the historic simulation as it is directly affected by historic data; but with the variance-covariance method, this can be made less likely by using alternative correlation/standard deviations to account for this.

References

1. Financial Engineering: Derivatives and Risk Management *by Keith Cuthbertson and Dirk Nitzsche*, Chapter 24
2. Simon Benninga and Zvi Wiener (1998) "Value-at-Risk", Vol 7 No.4
3. Thomas J. Linsmeier and Neil D. Pearson (July 1996) "Risk Measurement: An Introduction to Value at Risk", Pages 16-38.
4. http://www.investopedia.com/articles/04/092904.asp
5. http://www.exinfm.com/training/pdfiles/valueatrisk.pdf

Appendix

Test for normality and sample statistic

	BP	BRITISH_AIR	BARCLAYS
Mean	539.4612	284.1683	552.1551
Median	552.0000	275.5000	546.0000
Maximum	712.0000	577.5000	790.0000
Minimum	356.5000	86.00000	311.0000
Std. Dev.	78.12520	109.3518	95.21495
Skewness	-0.314973	0.506810	0.263116
Kurtosis	2.090565	2.741884	2.710027
Jarque-Bera	90.31380	80.73168	26.63913
Probability	0.000000	0.000000	0.000002
Sum	955385.8	503262.0	977866.8
Sum Sq. Dev.	10803277	21165339	16046621
Observations	1771	1771	1771

Appendix – Sample calculations

Historical method:

Date	BP	BRITISH AIRWAYS	BARCLAYS				Portfolio	Sorted	Quantile	
	BP.(P)	BAY(P)	BARC(P)							
06/05/2004	0.35	-4.02	-1.80	384.4	-1064.7	-1207.4	-1887.7	12872.2	100.00	
07/05/2004	0.15	-0.56	-0.99	164.2	-147.9	-664.6	-648.3	9149.5	99.90	
10/05/2004	-2.65	-4.96	-1.80	-2896.2	-1313.8	-1208.3	-5418.3	8805.9	99.79	
11/05/2004	0.67	3.74	1.93	729.7	991.1	1298.8	3019.6	8395.3	99.69	
12/05/2004	-1.02	-3.98	-0.90	-1115.2	-1056.0	-603.5	-2774.7	8393.2	99.59	
13/05/2004	1.80	1.19	0.76	1971.7	314.2	507.5	2793.4	7541.4	99.49	
14/05/2004	0.30	-3.61	-0.40	332.0	-957.5	-268.6	-894.1	7516.5	99.38	
17/05/2004	-0.45	-1.42	-0.95	-496.5	-375.9	-640.6	-1513.0	7403.2	99.28	
14/12/2007	1.56	-1.23	1.63	1709.4	-326.9	1090.9	2473.4	-3461.1	5.76	
17/12/2007	-1.54	-5.54	-2.54	-1683.1	-1468.8	-1705.0	-4856.8	-3469.4	5.65	
18/12/2007	-0.25	2.48	-1.74	-269.9	657.0	-1166.3	-779.1	-3475.0	5.55	
19/12/2007	-0.41	-0.89	-1.96	-450.9	-235.1	-1318.8	-2004.8	-3505.2	5.45	
20/12/2007	1.24	0.73	0.80	1358.4	194.1	538.1	2090.6	-3537.5	5.34	
21/12/2007	0.74	-0.24	0.30	805.1	-64.2	200.2	941.0	-3548.0	5.24	
24/12/2007	0.16	2.02	2.38	177.6	536.4	1596.6	2310.7	-3588.6	5.14	
27/12/2007	0.73	-1.83	-0.87	797.9	-483.7	-584.8	-270.7	-3657.9	5.03	**=> 95% VaR**
28/12/2007	-0.40	-0.57	-1.07	-440.1	-150.0	-721.1	-1311.1	-3661.4	4.93	-3659.08
31/12/2007	-0.65	0.73	-0.49	-706.9	193.9	-331.3	-844.4	-3714.5	4.83	
02/01/2008	0.24	2.26	0.10	266.8	598.9	66.6	932.3	-3759.2	4.73	
03/01/2008	3.08	-1.97	0.79	3371.6	-522.9	532.2	3380.9	-3864.4	4.62	
15/02/2008	-1.86	-3.97	-3.50	-2036.7	-1051.8	-2348.6	-5437.1	-6255.9	1.43	
18/02/2008	2.71	2.79	7.60	2964.8	738.1	5103.1	8805.9	-6263.4	1.33	
19/02/2008	0.62	2.01	3.70	673.5	532.8	2480.7	3687.0	-6270.5	1.22	
20/02/2008	-2.53	-3.34	2.73	-2773.3	-885.6	1829.4	-1829.5	-6339.4	1.12	
21/02/2008	-1.17	-3.55	-1.43	-1275.5	-939.7	-958.9	-3174.2	-6353.3	1.02	**=> 99% VaR**
22/02/2008	-0.54	-3.86	-0.62	-595.6	-1023.0	-416.9	-2035.5	-6673.1	0.91	-6406.09
25/02/2008	2.65	6.50	5.63	2894.7	1722.8	3775.8	8393.2	-6974.8	0.81	
26/02/2008	0.71	2.06	2.17	778.0	547.1	1456.4	2781.4	-7029.2	0.71	
27/02/2008	-0.88	-2.46	0.39	-965.6	-652.6	259.2	-1359.0	-7280.5	0.61	
28/02/2008	-2.40	-4.24	-3.75	-2630.3	-1123.1	-2517.2	-6270.5	-7621.5	0.50	
29/02/2008	-0.36	-3.01	-4.65	-399.3	-798.5	-3118.2	-4316.0	-7733.7	0.40	
03/03/2008	-1.01	-2.33	-3.20	-1102.0	-617.5	-2144.9	-3864.4	-8793.3	0.30	
04/03/2008	-0.83	-0.40	-1.14	-910.8	-105.4	-762.8	-1779.0	-9019.8	0.19	
05/03/2008	2.05	5.79	-2.03	2245.1	1533.9	-1359.4	2419.7	-12475.1	0.09	

Variance-Covariance

Portfolio Variance 17126839

Portfolio S.D. 4138.459

	No of Shares	Price	Value	Value^2
BA	100	265.00	26,500	702,250,000

BP	200	547.00	109,400	11,968,360,000
Barclays	150	447.50	67,125	4,505,765,625
Total			203,025	

Correlation	BP	BA	Barclays
BP	1.0000	0.2689	0.4373
BA	0.2689	1.0000	0.5002
Barclays	0.4373	0.5002	1.0000

Var-Cov Matrix	BP	BA	Barclays
BP	0.000247908	0.000121	0.000138
BA	0.000120698	0.000813	0.000287
Barclays	0.000138331	0.000287	0.000404

Stand Deviation	BP	BA	Barclays
BP	0.015749572		
BA		0.028521	
Barclays			0.020098

Confidence	**VaR**
95%	**-6828.45666**
99%	**-9642.60849**

Monte Carlo Simulation

Listing 1. Monte Carlo procedure (Matlab)

```
% Risk Management - Monte Carlo Simulation
 function pVal = monteCarlo( d, ret, est, swe, sva, nit,  col )
% inputs:
% d - drawing histogram ( 't' - draw, 'n' - don't)
% ret - returns
% est - estimation period length
% vle - future VaR number of days
% col - confidence level
% nus - number of series
% nos - number of stocks
% swe - stocks weights
% sva - stocks value
% nit - number of iterations
% reuturns
% VaR
 ssi = size( ret, 1 );
nus = size( ret, 2);
sMn = repmat( mean( ret( ssi - est + 1 : end, :)), [est 1]);
sStd = repmat( std( ret( ssi - est + 1 : end, :)), [est 1]);;
```

```
sCor = corrcoef (ret( ssi - est + 1 : end, :));

pVal = [];
pVa  = sva;
% repeat number of times
for i = 1 : nit
  for j = 1 : nus
    sRet( :, j ) = sMn( :, j ) + sStd( :, j ).*randn(est, 1);
  end
    sRet =sRet*chol(sCor);
    pV = swe;
    pVal = [pVal ; (sRet * pV'+1)* pVa];
end
```

Listing 2. Main procedure for Monte Carlo

```
% % Monte Carlo simulation for VAR
 clear all;
 [FileName,PathName,FilterIndex] = uigetfile('*.xls')

[price, txt]=xlsread(strcat(PathName,FileName), -1);
esp = size(price,1)-1;
epp = size(txt, 1)-esp;
 comps = txt(1,2:end);
 date = datenum(txt(epp:end,1), 'dd/mm/yyyy');

ret = tick2ret( price(epp-1:end, :) );
est = size(ret,1)-1;
noSh = [200 100 150];
pWeights = [noSh./sum(noSh)];
portVal = noSh*(price(end,:))';
pCal = [];
pVal =  monteCarlo('t', ret, est, pWeights, portVal, 1000, 99);
%%
[m,c]=hist(pVal, 100);
hist(pVal, 100);

col = 95;
var95 = portVal - c(100-95);
col = 95;
var99 = portVal - c(100-99);
[var95 var99]
```